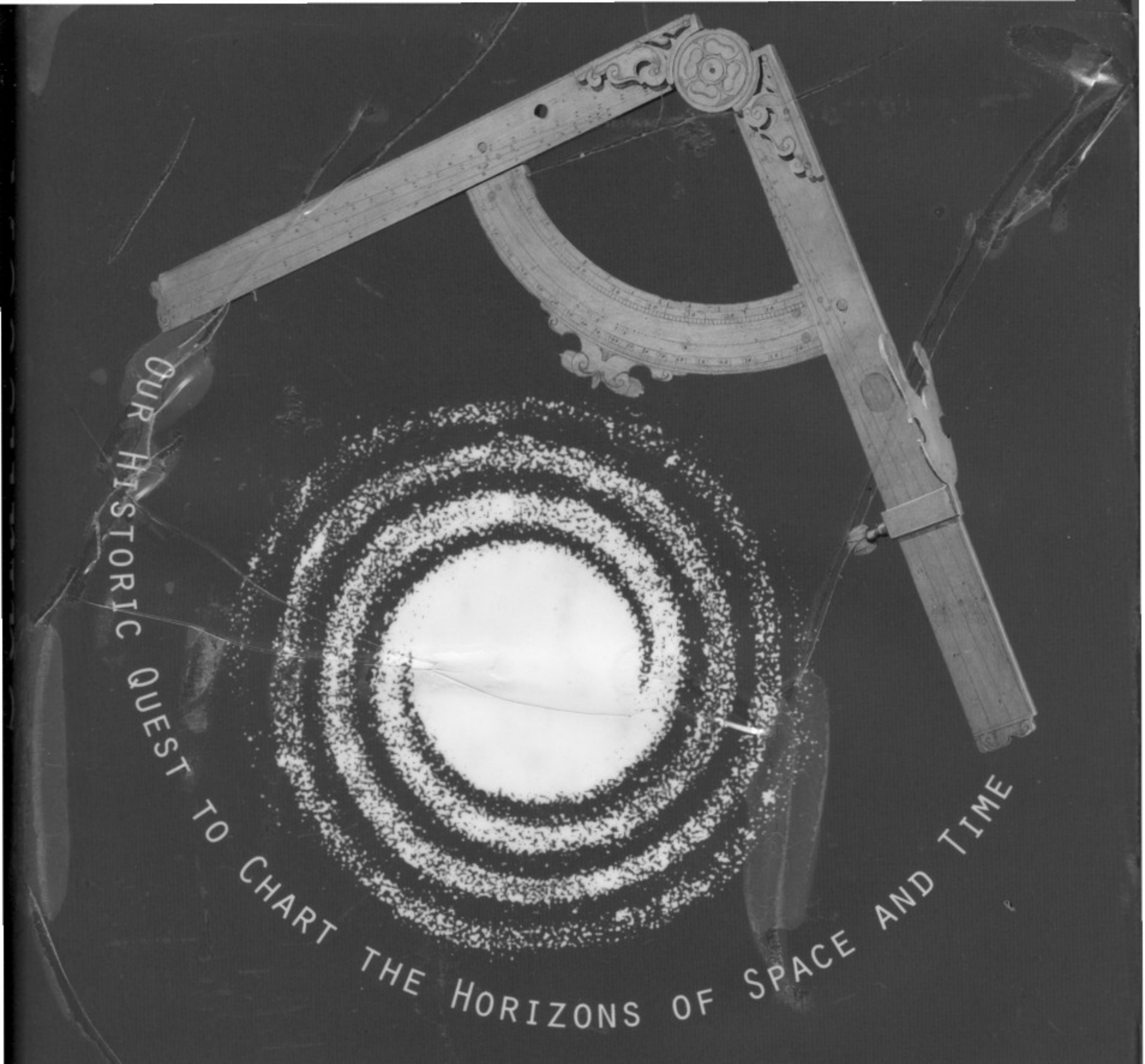

Our Historic Quest to Chart the Horizons of Space and Time

Measuring the Universe

Kitty Ferguson

A Sunlit Well at Syene

Eratosthenes, "son of Aglaos," was born not in Egypt or Greece but in the ancient city of Cyrene, on the northern coast of Africa in what is now Libya. Citizens of Crete and Santorini had founded Cyrene some 350 years earlier, and it had become one of the most cultured cities of the Hellenistic world, though still subordinate to the Egypt of the Ptolemys. Cyrene counted some distinguished figures among its citizenry. The founder of the Cyrenaic school, Aristippos, had been a pupil of Socrates. Aristippos's daughter, Arete, followed him as head of the school, and her son, Aristippos II, succeeded her. He was nicknamed "Metrodidactos," which translates as "mother-taught."

The date given for Eratosthenes's birth is the "126th Olympiad," referring to the Olympic games that took place every four years. In modern dating, that puts it between 276 and 273 B.C. He received most of his education in Athens at the feet of eminent scholars of the New Academy and the Lyceum. Plato and Aristotle had originally founded these schools (in Plato's day the New Academy had been simply the Academy), and though much had changed about them by the time Eratosthenes arrived, they were still the most prestigious educational institutions.

By the middle of the century, Eratosthenes had written a few philosophical and literary works, and some of these had come to the attention of Ptolemy III Euergetes. The "brain-drain" from Athens being in the general direction of Alexandria, Eratosthenes in about 244 B.C. agreed to move there and become a fellow of the museum and tutor to the prince, Philopator. It is perhaps not to Eratosthenes's credit that his pupil, though a patron of arts and learning, gained a reputation for dissipation and crimes that rivaled Nero's and Caligula's later in Rome.

In the course of time, Eratosthenes became a senior (alpha) fellow of the museum, and upon the death of the chief librarian he took over that post—an unparalleled vantage point from which to keep up with everything that was going on in the scholarly world.

Unfortunately, none of Eratosthenes's many works have survived except in fragments. It's not even certain that all the fragments are genuine. Most information about him comes from reports and references of others. However, there is enough to tell that Eratosthenes's measurement of the Earth and his motive for attempting it were rooted in his eclectic and far-ranging knowledge and interests. Eratosthenes was a man of the world, in the literal sense of those words. He refused to categorize people as Greeks opposed to barbarians, adopting a new Hellenistic global point of view that had begun to replace the more parochial mind-set of Greece in earlier centuries. He collected information about the people, products, and geography of far-flung areas. He wrote about the history of geographic measurement, recalling old ideas going back to Homer about the size, shape, and geographic layout of the Earth. In fact, he did nothing less than pull together virtually all the geographic knowledge that had been accumulating up to his own time.

Over the centuries, this material had taken a variety of forms. It came from traders, explorers, travelers—as well as mathematicians and philosophers—and it ranged from fantastic tales to more straightforward reporting, from speculation to measurements and estimates resting on what were probably recognized as shaky assumptions. Among the more reliable sources were eyewitness accounts of Alexander the Great's expeditions and the measurements and records of distances covered on those marches. There were itineraries of coastal voyages and maps and charts connected with them. There was a treatise on harbors by Timosthenes, the admiral of the Ptolemaic fleet, who also studied the winds. There was a book titled *On the Ocean* by the merchant sea captain Pytheas, who in about 320 B.C. sailed north along the coast of Spain and France and reached Cornwall, then continued all the way up to the Orkneys and the Shetlands to latitudes near those of the midnight sun. Pytheas took bearings throughout his voyage and recorded them in his book, which also had descriptive passages: "The barbarians showed us where the Sun keeps watch at night, for around these parts the night is exceedingly short, sometimes two and sometimes three hours, so that only a short interval passes after the Sun sets before it rises once more." Eratosthenes respected Pytheas's information, though many other scholars were contemptuous and disbelieving. Living as Eratosthenes did in Hellenistic Egypt, he may also have known of centuries-old and astoundingly accurate Egyptian geographic calculations.

Eratosthenes's expertise on longitude and latitude surpassed any other of his day or earlier. His predecessors had divided the map into zones. He took that work several steps farther by improving on a map devised about twenty-five years before his birth by a man named Dicaearchus of Messene, who had divided the known world by using two lines or bands that intersected one another—one running east-west, the other north-south. On Eratosthenes's revised map the two lines crossed at Rhodes, a little to the east of where Dicaearchus's lines had met. The horizontal line passed near Gibraltar (then known as the Pillars of Hercules), ran the length of the Mediterranean, and then followed the Taurus chain of mountains in southern Turkey. (Toros Daglari on later maps.) The path of that line is remarkably close to the course of the thirty-seventh parallel—an impressive achievement without the benefit of the mathematical and astronomical knowledge that would go into later mapmaking. It was not yet possible to calculate latitude with very great precision and virtually impossible to determine longitude (which would prove to be a problem in Eratosthenes's measurement of the Earth). Eratosthenes's vertical line, following the Nile, doesn't line up so perfectly with Rhodes on modern maps. He drew six more vertical lines at intervals between the western and eastern boundaries of the inhabited world, and six more horizontal lines at intervals between its northern and southern boundaries. In addition, he established and measured geographic zones, dividing the world horizontally between the tropical region, the temperate region, and the polar circles.

Eratosthenes was also well acquainted with state-of-the-art geometry, both from Euclid's brilliant summing up about twenty-five years before Eratosthenes's birth and from his association with Archimedes, an extraordinary genius and world-class eccentric. There is the familiar tale of Archimedes' solving a mathematical problem in his bath, leaping from the water, and running naked through the streets shouting "Eureka!" This avid mathematician eventually lost his life when Roman troops sacked Syracuse. Archimedes, so the story goes, was drawing a mathematical figure in the sand when a Roman soldier (who had missed hearing an order from his superiors to respect the person of this famous old man) asked him to pack up and move along. Archimedes unwisely told the soldier not to interrupt his thought process.

The Hellenistic world revered Archimedes as an inventor (though he himself dismissed those practical achievements as unworthy) and a useful man in wartime. According to legend, he destroyed a Roman fleet by using burning mirrors. The Middle Ages respected him as an engineer and a wizard and credited him with the invention of the Staff of Archimedes, a stick with a small flat disk that could be run up and down it, so that an observer holding it up to the Sun and noting the distance from disk to eye could derive the Sun's apparent diameter. Modern history and mathematics books recall Archimedes as a brilliant mathematician and geometer who contributed significantly to the understanding of circles and spheres. Archimedes was in the habit of sharing his discoveries and his methods with Eratosthenes and even dedicated his greatest work, *Method,* to him. Eratosthenes must have welcomed another scholar who was almost as eclectic as he was himself.

Eratosthenes's thoughts stretched to the horizon in all directions. Perhaps it follows that he would have longed to know not only what was beyond those horizons but how far "beyond" was. Mapping and systematizing things geographically was his bent. Would he not have been unusually curious about how large the total map was? How remarkable if it really should turn out to be, as Aristotle speculated, "a sphere of no great size"! Eratosthenes's thoughts often took a historical turn, and he was aware of previous attempts to measure the Earth or estimate that measurement. Would he not have wanted to try his own hand at it, using Euclid's and Archimedes' newer understanding of geometry?

There is still one circumstance to be mentioned—a simple, trivial matter, yet Eratosthenes's successful measurement of the circumference of the Earth would not have taken place without it. A happenstance, perhaps, that such a small gem of information reached the ears of this man who realized what it meant and what could be done with it. It is true that the fact that this snippet of news reached him did have something to do with the broadened mental horizons of the world, with improved communications from remote areas, with Eratosthenes's own world centering on northern Africa, and with his habit of keeping his ears and eyes open and wanting to know everything and anything. He was indeed the right man in the right time and place. Perhaps there was no other so likely to run across this back-page news and recognize its worth:

In a well located at Syene (near modern Aswan), on the day of the summer solstice, a shaft of sunlight penetrated all the way to the bottom of the well.

Eratosthenes knew that this signified that the Sun was shining directly down at Syene, not at an angle, which meant that Syene was on the "Tropic." A stick set up at noon at Syene on the day of the summer solstice would not cast a shadow. A stick set up at Alexandria (which he thought was the same longitude as Syene) *would.* Accordingly, Eratosthenes set up a stick at Alexandria on the day of the summer solstice and measured the angle of its shadow when that shadow was at its shortest.

Figure 1.1 shows the stick at Alexandria and its shadow and what "the angle of the shadow" means. If you draw a straight line from the point marked Alexandria (where the stick is casting a shadow) to the center of the Earth, and a second straight line from the point marked Syene (where the stick casts no shadow) to the center of the Earth, those lines will of course meet at the center of the Earth. The question is: What is the angle created by those two lines where they meet? The answer, as Eratosthenes knew, is that

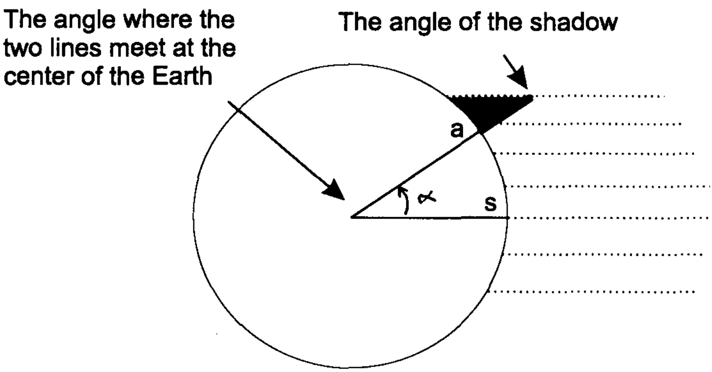

Figure 1.1 Because the Sun's rays are running parallel as they strike the Earth, if a line is drawn from Alexandria (a) where the stick casts a shadow, to the center of the Earth, and a second line from Syene (s) where there is no shadow, to the center of the Earth, the angle where those two lines meet will be the same as the angle of the shadow at Alexandria.

the angle at the center of the Earth and the angle of the shadow at Alexandria will be the same angle. Figure 1.2 illustrates Eratosthenes's measurement.

Eratosthenes found that the shadow angle at Alexandria was $7^1/_5$ degrees, and so he knew that the angle between the "Syene/Alexandria lines" (meeting at the center of the Earth) was also $7^1/_5$ degrees. A circle has 360 degrees, and it is a simple process to find out how many of the Syene-Alexandria angles ($7^1/_5$ degrees) it will take to make 360 degrees. Think of the cross section of the Earth as a pie and the two lines coming from Syene and Alexandria as cutting out a wedge of pie. How many wedges of that size can be cut from the whole pie? Divide 360 by $7^1/_5$, and it comes out to 50 wedges. Eratosthenes calculated that the distance between Syene and Alexandria at the surface of the Earth—at the pie-crust edge of the pie—is 5,000 stades. (There is a story that he sent a man to pace it off for him.) He multiplied 5,000 by 50 and concluded that the distance all the way around the Earth—the circumference of the Earth—is 250,000 stades. He later fine-tuned this to 252,000 stades.

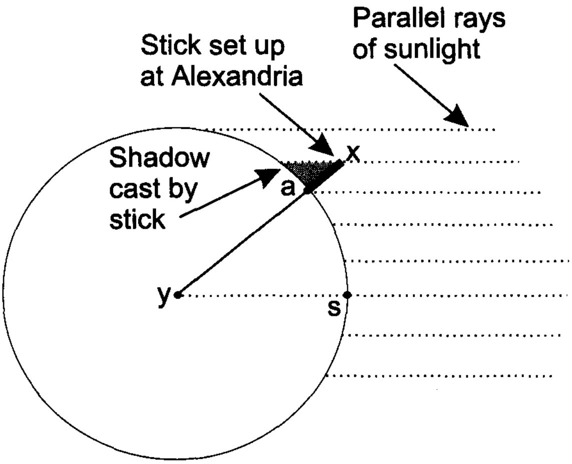

Figure 1.2 Because the sunlight shone all the way to the bottom of the well at Syene (s), Eratosthenes knew that the Sun was shining straight down on the Earth there. He set up a stick at Alexandria (a), where the Sun wasn't shining straight down, and he measured the angle (x) of the shadow cast by the stick. He knew that because the Sun's rays all run parallel as they strike the Earth, the angle (y) where a line drawn straight down from Alexandria and a line drawn straight down from Syene would meet at the center of the Earth would be the same angle as the angle of the shadow cast by the stick (x). If Syene is due south of Alexandria, then the distance between Syene and Alexandria must be the same fraction of the Earth's total circumference as the angle at x or y is of 360 degrees.

What is this odd unit of measurement, the stade? That question brings up a problem in evaluating Eratosthenes's result. Whether or not that result matches modern measurements for the circumference of the Earth depends on the length of stade he was using. If there are 157.5 meters in a stade, Eratosthenes's result comes to 24,608 miles (39,690 kilometers) for the circumference of the Earth. That is very near the modern calculation: 24,857 miles (40,009 kilometers) around the poles and 24,900 miles (40,079 kilometers) around the equator. After he found the circumference, Eratosthenes calculated the diameter of the Earth as 7,850 miles (12,631 kilometers), close to today's mean value of 7,918 miles (12,740 kilometers).

Another way of figuring a stade was as one-eighth or one-tenth of a Roman mile, and that would make Eratosthenes's result too large by modern standards. There was one additional small difficulty. Eratosthenes assumed that Syene lay on the same line of longitude as Alexandria. Actually, it did not.

But this is nit-picking! No apology need be made for Eratosthenes. First of all, he arguably came astonishingly near to matching the modern measurement. Second, he found a way to solve this problem by the imaginative use of geometry. His *method* was ingenious and correct. If the numerical result is a little fuzzy because of a lack of agreement about the length of a stade and the impossibility of determining longitude precisely, that does not detract from the brilliance of his achievement or of the intellectual leap involved in recognizing that measuring the Earth's circumference *could* be done and *how* it could be done.

Eratosthenes's curiosity went beyond the Earth. He also considered the astronomical questions of his day. When it came to measuring the distances to the Sun and the Moon, he must have realized that he had no tool at his fingertips to equal the news about the well in Syene. Nevertheless, he gave it a try, with far less success than he had in measuring the Earth's circumference.